

```

> restart
> Ecua := y'' + 6·y' + 13·y = exp(-3 x)cos(2 x)
      Ecua :=  $\frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 13 y(x) = e^{-3x} \cos(2x)$  (1)
=
> EcuaHom := lhs(Ecua) = 0
      EcuaHom :=  $\frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 13 y(x) = 0$  (2)
=
> Q := rhs(Ecua)
      Q :=  $e^{-3x} \cos(2x)$  (3)
=
Método de Parámetros Variables
> EcuaCarac := m^2 + 6·m + 13 = 0
      EcuaCarac :=  $m^2 + 6m + 13 = 0$  (4)
=
> Raiz := solve(EcuaCarac)
      Raiz :=  $-3 + 2I, -3 - 2I$  (5)
=
> yy[1] := exp(Re(Raiz[1])·x)·cos(Im(Raiz[1])·x); yy[2] := exp(Re(Raiz[1])·x)
  ·sin(Im(Raiz[1])·x);
      yy1 :=  $e^{-3x} \cos(2x)$ 
      yy2 :=  $e^{-3x} \sin(2x)$  (6)
=
> SolGralHom := y(x) = C[1]·yy[1] + C[2]·yy[2]
      SolGralHom :=  $y(x) = C_1 e^{-3x} \cos(2x) + C_2 e^{-3x} \sin(2x)$  (7)
=
> SolGralNoHom := y(x) = AA·yy[1] + BB·yy[2]
      SolGralNoHom :=  $y(x) = AA e^{-3x} \cos(2x) + BB e^{-3x} \sin(2x)$  (8)
=
> with(linalg) :
> WW := wronskian([yy[1], yy[2]], x)
      WW :=  $\begin{bmatrix} e^{-3x} \cos(2x) & e^{-3x} \sin(2x) \\ -3 e^{-3x} \cos(2x) - 2 e^{-3x} \sin(2x) & -3 e^{-3x} \sin(2x) + 2 e^{-3x} \cos(2x) \end{bmatrix}$  (9)
=
> BBB := array([0, Q])
      BBB :=  $\begin{bmatrix} 0 & e^{-3x} \cos(2x) \end{bmatrix}$  (10)
=
> ParaVar := simplify(linsolve(WW, BBB))
      ParaVar :=  $\begin{bmatrix} -\frac{\cos(2x) \sin(2x)}{2} & \frac{\cos(2x)^2}{2} \end{bmatrix}$  (11)
=
> Aprima := ParaVar[1]; Bprima := ParaVar[2]
      Aprima :=  $-\frac{\cos(2x) \sin(2x)}{2}$ 
      Bprima :=  $\frac{\cos(2x)^2}{2}$  (12)
=
> AA := int(Aprima, x); BB := int(Bprima, x)

```

$$AA := \frac{\cos(2x)^2}{8}$$

$$BB := \frac{\cos(2x) \sin(2x)}{8} + \frac{x}{4} \quad (13)$$

$$\begin{aligned} &> \text{SolGralNoHom} := y(x) = \text{simplify}((AA + C[1]) \cdot yy[1] + (BB + C[2]) \cdot yy[2]) \\ &\text{SolGralNoHom} := y(x) = \frac{\left(\sin(2x) (x + 4 C_2) + 4 \left(C_1 + \frac{1}{8} \right) \cos(2x) \right) e^{-3x}}{4} \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{SolGralFinal} := y(x) = C[1] \cdot \exp(-3x) \cdot \cos(2x) + C[2] \cdot \exp(-3x) \cdot \sin(2x) \\ &\quad + \frac{x \cdot \exp(-3x) \cdot \sin(2x)}{4} \\ &\text{SolGralFinal} := y(x) = C_1 e^{-3x} \cos(2x) + C_2 e^{-3x} \sin(2x) + \frac{e^{-3x} \sin(2x) x}{4} \end{aligned} \quad (15)$$

> Ecua

$$\frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 13 y(x) = e^{-3x} \cos(2x) \quad (16)$$

$$\begin{aligned} &> \text{Comprobar} := \text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolGralFinal}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0)) \\ &\text{Comprobar} := 0 = 0 \end{aligned} \quad (17)$$

$$\begin{aligned} &> \text{CondIni} := y(0) = 10, D(y)(0) = -5 \\ &\text{CondIni} := y(0) = 10, D(y)(0) = -5 \end{aligned} \quad (18)$$

$$\begin{aligned} &> \text{EcuaUno} := \text{simplify}(\text{subs}(x = 0, \text{rhs}(\text{SolGralFinal})) = 10) \\ &\text{EcuaUno} := C_1 = 10 \end{aligned} \quad (19)$$

$$\begin{aligned} &> \text{EcuaDos} := \text{simplify}(\text{subs}(x = 0, \text{rhs}(\text{diff}(\text{SolGralFinal}, x))) = -5) \\ &\text{EcuaDos} := -3 C_1 + 2 C_2 = -5 \end{aligned} \quad (20)$$

$$\begin{aligned} &> \text{ParaPara} := \text{solve}(\{\text{EcuaUno}, \text{EcuaDos}\}) \\ &\text{ParaPara} := \left\{ C_1 = 10, C_2 = \frac{25}{2} \right\} \end{aligned} \quad (21)$$

$$\begin{aligned} &> \text{SolPart} := \text{subs}(C[1] = \text{rhs}(\text{ParaPara}[1]), C[2] = \text{rhs}(\text{ParaPara}[2]), \text{SolGralFinal}) \\ &\text{SolPart} := y(x) = 10 e^{-3x} \cos(2x) + \frac{25 e^{-3x} \sin(2x)}{2} + \frac{e^{-3x} \sin(2x) x}{4} \end{aligned} \quad (22)$$

$$\begin{aligned} &> \text{ComprobarDos} := \text{simplify}(\text{subs}(x = 0, \text{SolPart})) \\ &\text{ComprobarDos} := y(0) = 10 \end{aligned} \quad (23)$$

$$\begin{aligned} &> \text{ComprobarTres} := D(y)(0) = \text{simplify}(\text{subs}(x = 0, \text{rhs}(\text{diff}(\text{SolPart}, x)))) \\ &\text{ComprobarTres} := D(y)(0) = -5 \end{aligned} \quad (24)$$

$$\begin{aligned} &> \text{ComprobarCuatro} := \text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolPart}), \text{lhs}(\text{Ecua}) - \text{rhs}(\text{Ecua}) = 0)) \\ &\text{ComprobarCuatro} := 0 = 0 \end{aligned} \quad (25)$$

>
> restart

[Método del Operador Diferencial

$$\begin{aligned} > \text{Ecua} := \frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 13 y(x) = e^{-3x} \cos(2x) \\ & \text{Ecua} := \frac{d^2}{dx^2} y(x) + 6 \frac{d}{dx} y(x) + 13 y(x) = e^{-3x} \cos(2x) \end{aligned} \quad (26)$$

$$\begin{aligned} > \text{SolNoHom} := y(x) = A \cdot x \cdot \exp(-3x) \cdot \cos(2x) + B \cdot x \cdot \exp(-3x) \cdot \sin(2x) \\ & \text{SolNoHom} := y(x) = A x e^{-3x} \cos(2x) + B x e^{-3x} \sin(2x) \end{aligned} \quad (27)$$

$$\begin{aligned} > \text{Parametros} := \text{eval}(\text{subs}(y(x) = \text{rhs}(\text{SolNoHom}), \text{Ecua})) \\ & \text{Parametros} := -4 A e^{-3x} \sin(2x) + 4 B e^{-3x} \cos(2x) = e^{-3x} \cos(2x) \end{aligned} \quad (28)$$

$$\begin{aligned} > \text{Para}[1] := -4 A = 0 \\ & \text{Para}_1 := -4 A = 0 \end{aligned} \quad (29)$$

$$\begin{aligned} > \text{Para}[2] := 4 \cdot B = 1 \\ & \text{Para}_2 := 4 B = 1 \end{aligned} \quad (30)$$

$$\begin{aligned} > \text{ParaDos} := \text{solve}(\{\text{Para}[1], \text{Para}[2]\}) \\ & \text{ParaDos} := \left\{ A = 0, B = \frac{1}{4} \right\} \end{aligned} \quad (31)$$

$$\begin{aligned} > \text{SolNoHomFinal} := \text{subs}(A = \text{rhs}(\text{ParaDos}[1]), B = \text{rhs}(\text{ParaDos}[2]), \text{SolNoHom}) \\ & \text{SolNoHomFinal} := y(x) = \frac{x e^{-3x} \sin(2x)}{4} \end{aligned} \quad (32)$$

> restart

$$\begin{aligned} > \text{Ecua} := y'' + 4 \cdot y = \sin(x) \cdot \sin(2x) \\ & \text{Ecua} := \frac{d^2}{dx^2} y(x) + 4 y(x) = \sin(x) \sin(2x) \end{aligned} \quad (33)$$

$$\begin{aligned} > \text{EcuaHom} := \text{lhs}(\text{Ecua}) = 0 \\ & \text{EcuaHom} := \frac{d^2}{dx^2} y(x) + 4 y(x) = 0 \end{aligned} \quad (34)$$

$$\begin{aligned} > Q := \text{rhs}(\text{Ecua}) \\ & Q := \sin(x) \sin(2x) \end{aligned} \quad (35)$$

$$\begin{aligned} > \text{EcuaCarac} := m^2 + 4 = 0 \\ & \text{EcuaCarac} := m^2 + 4 = 0 \end{aligned} \quad (36)$$

> with(linalg):

$$\begin{aligned} > \text{Raiz} := \text{solve}(\text{EcuaCarac}) \\ & \text{Raiz} := 2 I, -2 I \end{aligned} \quad (37)$$

$$\begin{aligned} > yy[1] := \cos(\text{Im}(\text{Raiz}[1]) \cdot x); yy[2] := \sin(\text{Im}(\text{Raiz}[1]) \cdot x) \\ & yy_1 := \cos(2x) \\ & yy_2 := \sin(2x) \end{aligned} \quad (38)$$

$$\begin{aligned} > \text{SolGralHom} := y(x) = C[1] \cdot yy[1] + C[2] \cdot yy[2] \\ & \text{SolGralHom} := y(x) = \cos(2x) C_1 + \sin(2x) C_2 \end{aligned} \quad (39)$$

$$> \text{WWSolFinal} := \text{wronskian}([yy[1], yy[2]], x)$$

$$WW := \begin{bmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{bmatrix} \quad (40)$$

> $BB := \text{array}([0, Q])$

$$BB := \begin{bmatrix} 0 & \sin(x) & \sin(2x) \end{bmatrix} \quad (41)$$

> $ParaVar := \text{simplify}(\text{linsolve}(WW, BB))$

$$ParaVar := \begin{bmatrix} -\frac{\sin(2x)^2 \sin(x)}{2} & \frac{\cos(2x) \sin(x) \sin(2x)}{2} \end{bmatrix} \quad (42)$$

> $Aprima := ParaVar[1]; Bprima := ParaVar[2]$

$$Aprima := -\frac{\sin(2x)^2 \sin(x)}{2}$$

$$Bprima := \frac{\cos(2x) \sin(x) \sin(2x)}{2} \quad (43)$$

> $A := \text{int}(Aprima, x); B := \text{int}(Bprima, x)$

$$A := \frac{\cos(x)}{4} + \frac{\cos(3x)}{24} - \frac{\cos(5x)}{40}$$

$$B := \frac{\sin(3x)}{24} - \frac{\sin(5x)}{40} \quad (44)$$

> $SolGral := y(x) = \text{simplify}((A + C[1]) \cdot yy[1] + (B + C[2]) \cdot yy[2])$

$$SolGral := y(x) = \frac{2 \cos(x)^3}{5} + 2 C_1 \cos(x)^2 + 2 \sin(x) \cos(x) C_2 - \frac{2 \cos(x)}{15} - C_1 \quad (45)$$

> $SolFinal := y(x) = \frac{2 \cos(x)^3}{5} + C_1 \cdot (\cos(2x)) + C_2 \cdot \sin(2x) - \frac{2 \cos(x)}{15}$

$$SolFinal := y(x) = \frac{2 \cos(x)^3}{5} + \cos(2x) C_1 + \sin(2x) C_2 - \frac{2 \cos(x)}{15} \quad (46)$$

> $Ecua$

$$\frac{d^2}{dx^2} y(x) + 4 y(x) = \sin(x) \sin(2x) \quad (47)$$

> $Comprobar := \text{simplify}(\text{eval}(\text{subs}(y(x) = \text{rhs}(SolFinal), \text{lhs}(Ecua) - \text{rhs}(Ecua) = 0)))$

$$Comprobar := 0 = 0 \quad (48)$$

>
>
>
>